Student Number:
Teacher:

St George Girls High School

Mathematics Advanced

2021 Trial Higher School Certificate Examination

General Instructions

- Reading time – 10 minutes
- Working Time – 3 hours
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in **Section I,** use the multiple-choice answer sheet provided.
- For questions in **Section II:**
  - Answer the questions in the space provided in the 6 booklets.
  - Show relevant mathematical reasoning and/or calculations.
  - Extra writing space is provided at the back of the booklets. If you use this space, clearly indicate which question you are answering.
  - Marks may not be awarded for incomplete or poorly presented solutions

Total marks: 100

Section I – 10 marks (pages 3 – 7**)**
- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 – 34**)**
- Attempt Questions 11 – 32
- Allow about 2 hour and 45 minutes for this section

Q1 – Q10	/10
Q11 – Q15	/15
Q16 – Q20	/15
Q21 – Q23	/15
Q24 – Q26	/17
Q27 – Q30	/15
Q31 – Q33	/13
Total	**/100**
	%

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1- 10.

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1 Which interval gives the range of the function $y = \frac{\sqrt{25-x^2}}{5}$?

A. $[0, 5]$

B. $[0, 1]$

C. $[-5, 5]$

D. $[-1, 1]$

2 An arithmetic sequence has a first term of 8 and a common difference of $-\frac{4}{5}$.
Which is closest to the value of the 27^{th} term of the sequence?

A. 53.9

B. 0.020

C. -12.80

D. -35.00

3 What is the equation of the tangent to the curve $y = \sin x$ at the point where $x = \pi$.

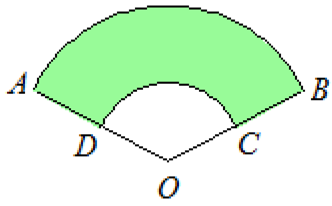
A. $y = x - \pi$

B. $y = \cos x$

C. $y = \sin x$

D. $y = -x + \pi$

4 A car windscreen wiper traces out the area $ABCD$ where AB and CD are arcs of circles with a centre O and radii 50 cm and 30 cm respectively.
Angle AOB measures 135°.



What is the area of the shaded region $ABCD$ to the nearest cm squared?

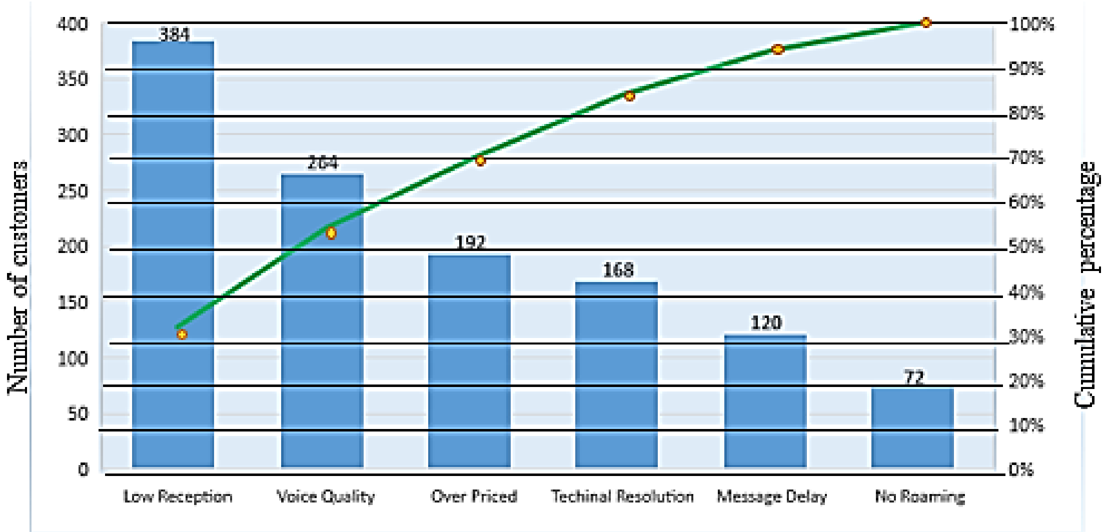
A. 600 cm^2

B. 1257 cm^2

C. 1885 cm^2

D. 96000 cm^2

5 Pronto Mobiles manager collected data given by customers related to the reasons for being unhappy with his company. The Pareto chart shows the data collected.



What percentage of the customers were unhappy because the service was Over Priced?

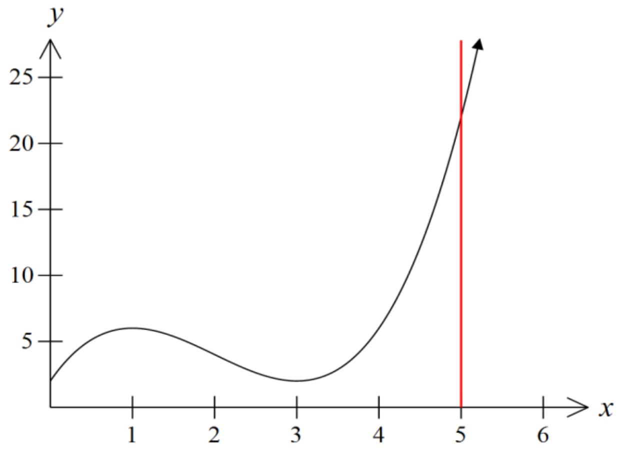
A. 70%

B. 16%

C. 54%

D. 48%

6 The diagram below shows the graph of $y = x^3 - 6x^2 + 9x + 2$ and the line $x = 5$.



What is the value of the area bounded by the x-axis and the curve $y = x^3 - 6x^2 + 9x + 2$ between $0 \leq x \leq 5$?

A. $A = 22.00$ square units

B. $A = 25.25$ square units

C. $A = 27.00$ square units

D. $A = 28.75$ square units

7 What is the amplitude, phase and period for the function $f(x) = \sin\left(\frac{x+\pi}{3}\right)$?

A. Amplitude 3, phase $\frac{\pi}{3}$ and period $\frac{\pi}{2}$

B. Amplitude 3, phase π and period $\frac{\pi}{2}$

C. Amplitude 4, phase $\frac{\pi}{3}$ and period 6π

D. Amplitude 4, phase π and period 6π

8 Let $g(x) = 1 - f(x)$. If $\int_1^4 f(x)dx = 7$, what is the value of $\int_1^4 g(x)dx$?

A. -6

B. -4

C. 8

D. 10

9 The curve $y = 2x^3 - ax^2 + 6x$ has a point of inflection point at $\mathrm{x} = 4$.
What is the value of a?

A. -18

B. -24

C. 24

D. 18

10 The discrete random variable X has the following probability distribution.

x	-1	0	1	2	3
$P(X = x)$	$\frac{1}{10}$	a	b	0.3	$2b$

What are the values of a and b if the expected value $E(X)$ is 1.2?

A. $a = 0.3$ and $b = 0.1$

B. $a = 0.1$ and $b = 0.3$

C. $a = 0.3$ and $b = 0.3$

D. $a = 0.1$ and $b = 0.1$

End of Section I

Mathematics Advanced

Section II Answer Booklet 1

Student Number:
Teacher:

Section II
90 marks
Attempt Questions 11 – 32
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Question 11 – 15 (15 marks)
Booklet 2 – Attempt Question 16 – 20 (15 marks)
Booklet 3 – Attempt Question 21 – 23 (15 marks)
Booklet 4 – Attempt Question 24 – 26 (17 marks)
Booklet 5 – Attempt Question 27 – 30 (15 marks)
Booklet 6 – Attempt Question 31 – 32 (13 marks)

Instructions

- Write your Teacher's Name and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 11 of Booklet 1. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (1 mark)

Find the derivative of $f(x) = 2x^2 - \frac{3}{x}$ 1

Question 12 (3 marks)

Calculate the sum of the arithmetic series $6 + 14 + 22 + \cdots + 958$. 3

Question 13 (4 marks)

Find

a) $\frac{d}{dx}(\ln\sqrt{3-x})$ 2

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b) $\frac{d}{dx}(\frac{\tan 3x}{x^3})$ 2

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Question 14 (2 marks)

Prove that $\cos\theta\cot\theta + \sin\theta = \operatorname{cosec}\theta$. 2

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Question 15 (5 marks)

Basketball and tennis are two sports students may choose to play.
For a group of 44 students, the following is known.

- 8 students play both.
- 20 students play tennis.
- 6 play neither.

a) A student is chosen at random. By using a Venn diagram or otherwise, find the probability that a student plays only basketball. 2

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b) A student is chosen at random. Given the student does not play basketball, what is the probability that the student plays tennis? 1

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c) Two students from this group are randomly selected after each other. What is the probability that the first only plays basketball and the second plays only tennis? 2

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Question 16 (2 marks)

Given that $0 < x < 4$ and $\log_5(4 - x) - \log_5 3x = 1$, find the value of x. 2

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Question 17 (3 marks)

Evaluate $\int_2^3 \frac{x-3}{x^2-6x+5}\,dx$ 3

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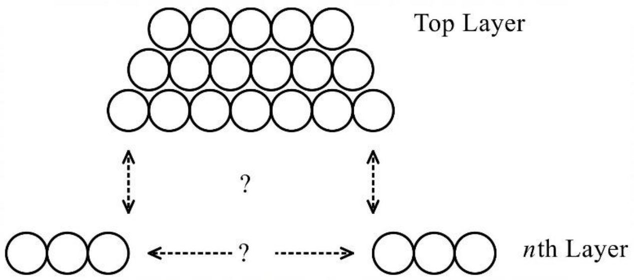
Question 18 (4 marks)

Marley works in a local fruit and vegetable store, and he is making a stack of oranges against a sloping display panel.

The oranges are stacked in layers, as shown, where each layer contains one orange less than the layer below it.



When he has finished, there are five oranges in the top layer, six in the next and so on.

There are n layers altogether.

a) Show there are $\frac{1}{2}n(n+9)$ oranges in the stack. 2

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b) If Lachlan has 300 oranges to create his display, how many full rows can he create, if the top row still contains five oranges? 2

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Question 19 (3 marks)

a) Differentiate $(1 + \ln x)^4$. 2

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b) Hence, find $\int \frac{(1 + \ln x)^3}{x}\, dx$. 1

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Question 20 (3 marks)

Solve the equation $2\sin^2\left(\frac{x}{3}\right) = 1$ where $[-\pi, \pi]$. 3

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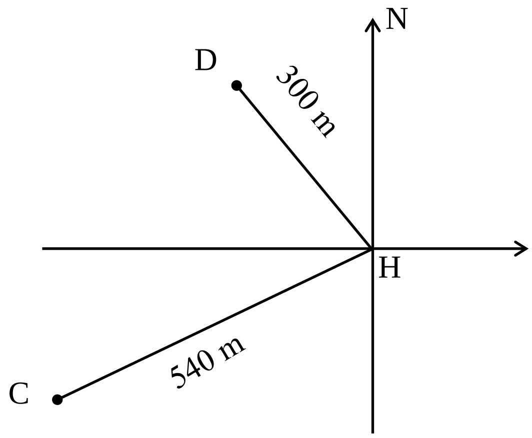
Question 21 (5 marks)

Layla and her brother Mike are at their home H. Layla walked 300 m on a bearing 320° to her dancing school D and Mike walked 540 m on a bearing 245° to his sports club C.



a) Show that the angle DHC is 75°. 1

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b) Find the distance DC, correct to 2 decimal places. 2

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c) Find the bearing of the sports club C from the dancing school D correct to the nearest degree. 2

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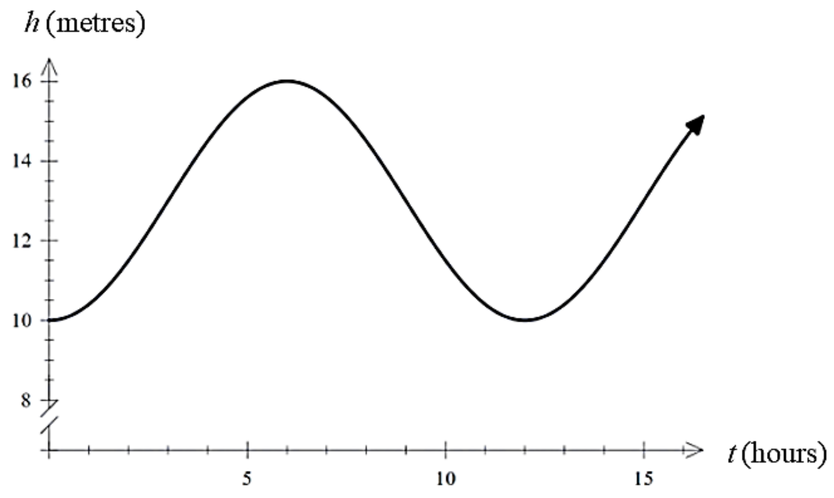
Question 22 (7 marks)

The rise and fall in sea level in Korsachov, due to tides, can be modelled by the cosine function below:

$$\boldsymbol{h(t) = A\cos(bt) + d}$$



At 8am it is low tide, and the channel is 10m deep. At 2pm it is high tide, and the channel is 16m deep. A ship needs at least 11.5m of water to sail.

a) Find the value of $A, b,$ and d. 3

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b) Between what times in the first day can the ship sail? 2

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c) What is the rate of change of the water level at 12.00 p.m.? 2

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Question 23 (3 marks)

Consider the geometric series

$$1 + \left(\sqrt{7} - 2\right) + (\sqrt{7} - 2)^2 + \cdots$$

a) Show with calculations why this geometric series has a limiting sum. 1

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b) Find the exact value of the limiting sum with a rational denominator. 2

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Question 24 (8 marks)

A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T°C, t minutes after it enters the liquid is given by,

$$T = 400e^{-0.05t} + 25, \quad t \geq 0.$$

a) Find the initial temperature of the ball as it enters the liquid. 1

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b) Find the value of t for which $T = 200$, giving you answer to 3 decimal places. 2

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c) Find the rate at which the temperature of the ball is decreasing after 50 minutes, giving your answer to 3 decimal places. 2

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d) As it cools the ball will eventually approach what temperature? **1**

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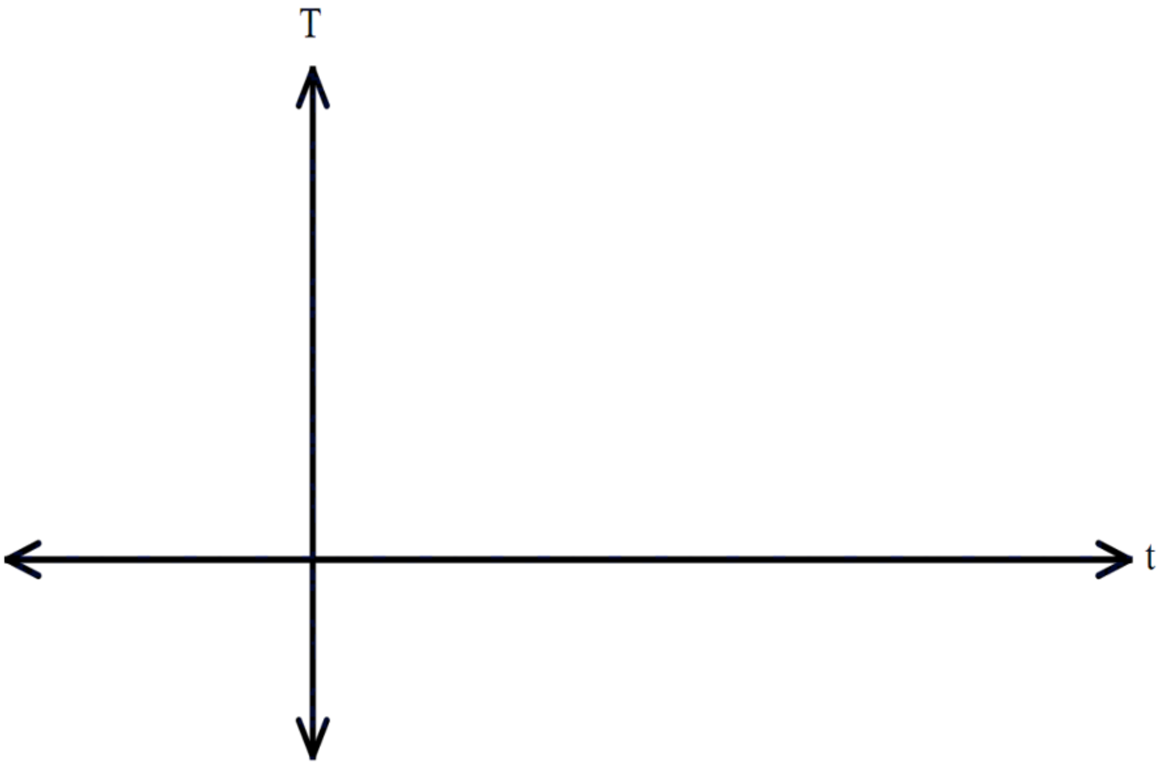
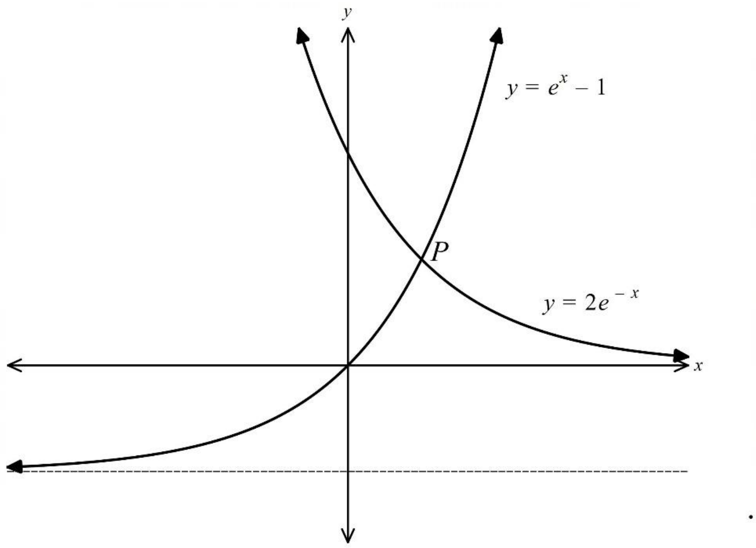
e) Sketch the graph of T. 2



Question 25 (6 marks)

Two curves $y = 2e^{-x}$ and $y = e^x - 1$ intersect at a point P.



a) Show algebraically that the coordinates of P are $(\ln 2, 1)$. 3

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b) Find the area bounded by the two curves and the y-axis. 3

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Question 26 (3 marks)

The parabola $y = x^2 - 2x$ is dilated horizontally by factor of 2 then translated vertically up by 3 to give a new parabola.

a) Sketch the transformed new parabola showing its vertex. 2

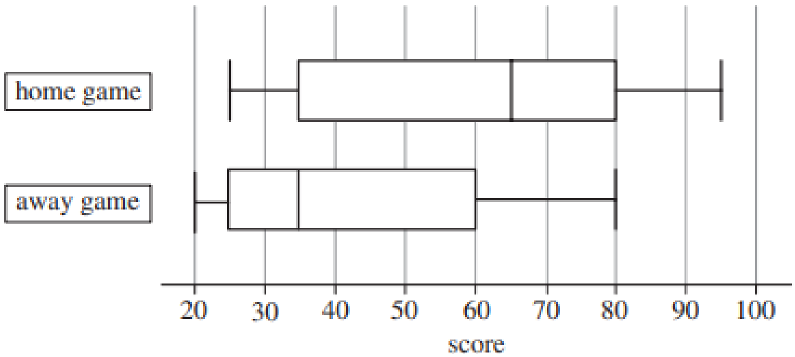
b) Find the equation of the new transformed parabola. 1

Question 27 (5 marks)

A local basketball team played 23 home games and 23 away games.

The home game (H) scores are the scores that the team made during games at their own basketball court; the away game (A) scores are the scores that the team made during games at opponents' courts.

The scores from each game are represented in the box-plot.



a) What was the team's lowest home game score? 1

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b) What percentage of the team's away game scores were less than 35? 1

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c) Compare and contrast the distributions of the team's home and away game scores. 3
In your answer, comment on the skewness of the distributions, measures of central tendency and spread.

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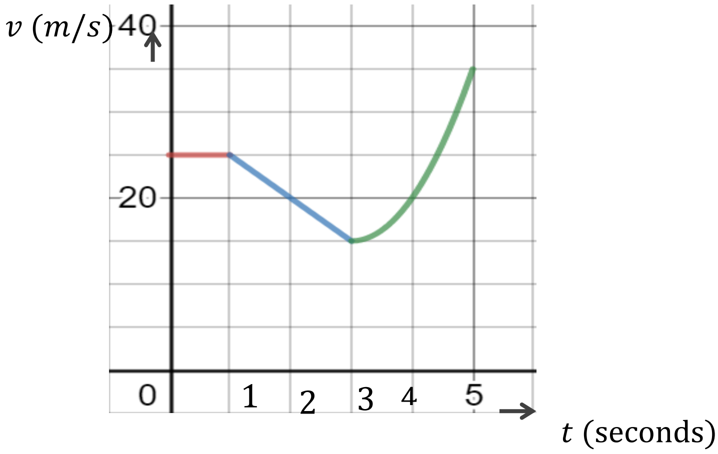
Question 28 (4 marks)

The graph shows the velocity (in metres per second) of a particle for 5 seconds.



The distance covered by the particle over 5 seconds is given by $\int_0^5 v(t)\ dt$.

Use the trapezoidal rule and the velocity at each of the six times values from $t = 0$ to $t = 5$ to find the approximate distance covered.

Give your answer correct to one decimal place. 2

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b) Is the distance travelled between 3 seconds and 5 seconds an overestimate or an underestimate of the actual distance covered by the particle? 2
Justify your answer.

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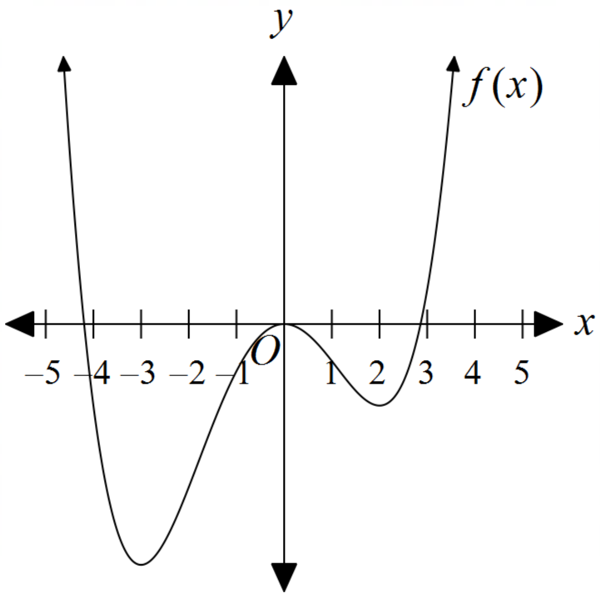
Question 29 (4 marks)

The sum of the first two terms of an infinite geometric series is 45. The third term in the series is 12. 4

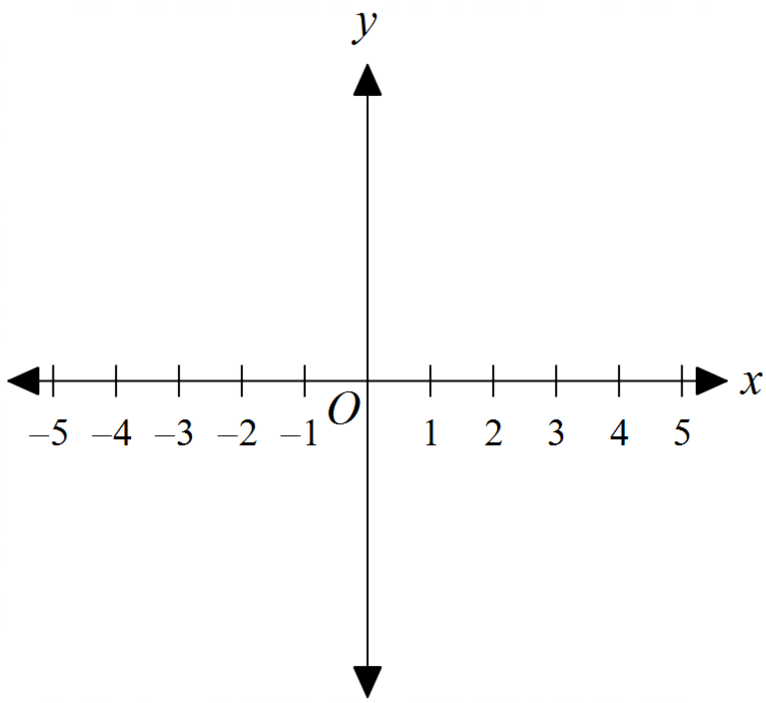
Show that there are two possible series and find the first term and common ratio for each case.

Question 30 (2 marks)

The diagram shows the graph of $f(x)$.



Sketch the graph of $g'(x)$, showing approximate $x-$intercepts and the behaviour as $x \to \pm\infty$. 2



Question 31 (6 marks)

A particle is moving in a straight line with a velocity given by $\frac{dx}{dt} = 8\cos\left(2t - \frac{\pi}{2}\right)$,

where x is the displacement from the origin in metres and t is time measured in seconds. The particle is initially at rest 4 m to the right of the origin.

a) Show that the displacement of the particle is given by $x = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$. 2

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b) Find the first time when the particle comes to rest again. 2

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c) By finding a function for the acceleration, describe the motion of the particle directly after $\frac{\pi}{2}$ s and before it next comes to rest. 2

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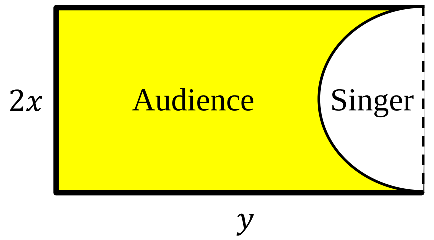
Question 32 (7 marks)

A concert hall is to be built in the shape of a rectangle with width $2x$ metres and length y metres.



The hall is to be divided into two parts. One section is for the audience and the other section is for the singer as shown in the diagram above.

The audience section is required to have an area of 500 m^2 and its perimeter should be as small as possible.

Let P be the perimeter of the audience section of the concert hall.

a) Show that the perimeter of the audience section of the concert hall is given by 3

$$P = \left(2 + \frac{3\pi}{2}\right)x + \frac{500}{x}.$$

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b) Find the radius of the semicircle of the hall where the singer stands that gives the smallest possible perimeter for the audience section. 4

Show, by mathematical reasoning, why this radius gives the required minimum perimeter.

END OF EXAMINATION

Student Number:	
Teacher:	Solutions

St George Girls High School

Mathematics Advanced

2021 Trial Higher School Certificate Examination

General Instructions

- Reading time – 10 minutes
- Working Time – 3 hours
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in **Section I,** use the multiple-choice answer sheet provided.
- For questions in **Section II:**
  - Answer the questions in the space provided in the 6 booklets.
  - Show relevant mathematical reasoning and/or calculations.
  - Extra writing space is provided at the back of the booklets. If you use this space, clearly indicate which question you are answering.
  - Marks may not be awarded for incomplete or poorly presented solutions

Total marks: 100

Section I – 10 marks (pages 3 – 7**)**

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 – 34**)**

- Attempt Questions 11 – 32
- Allow about 2 hour and 45 minutes for this section

Q1 – Q10	/10
Q11 – Q15	/15
Q16 – Q20	/15
Q21 – Q23	/15
Q24 – Q26	/17
Q27 – Q30	/15
Q31 – Q33	/13
Total	**/100**
	%

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1- 10.

---

1 Which interval gives the range of the function $y = \frac{\sqrt{25-x^2}}{5}$?

A. $[0, 5]$

B. $[0, 1]$

C. $[-5, 5]$

D. $[-1, 1]$

D: $[-5, 5]$
$\therefore$ R: $[0, 1]$

2 An arithmetic sequence has a first term of 8 and a common difference of $-\frac{4}{5}$.
Which is closest to the value of the 27th term of the sequence?

A. 53.9

B. 0.020

C. -12.80

D. -35.00

$a = 8$, $d = -\frac{4}{5}$
$T_{27} = 8 + 26 \times \left(-\frac{4}{5}\right)$
$= -12.80$

3 What is the equation of the tangent to the curve $y = \sin x$ at the point where $x = \pi$.

A. $y = x - \pi$

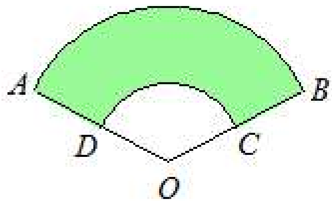
B. $y = \cos x$

C. $y = \sin x$

D. $y = -x + \pi$

$y' = \cos x$
at $x = \pi$
$m_T = \cos \pi$
$= -1$
when $x = \pi$, $y = \sin \pi = 0$
$\therefore$ $y - 0 = -1(x - \pi)$
$y = -x + \pi$

4 A car windscreen wiper traces out the area $ABCD$ where AB and CD are arcs of circles with a centre O and radii 50 cm and 30 cm respectively.
Angle AOB measures 135°.



What is the area of the shaded region $ABCD$ to the nearest cm squared?

$$A = \frac{1}{2}\theta(R^2 - r^2)$$
$$= \frac{1}{2} \times \frac{3\pi}{4}(50^2 - 30^2)$$
$$= 1884.955$$

A. 600 cm^2

B. 1257 cm^2

C. 1885 cm^2

D. 96000 cm^2

5 Pronto Mobiles manager collected data given by customers related to the reasons for being unhappy with his company. The Pareto chart shows the data collected.

What percentage of the customers were unhappy because the service was Over Priced?

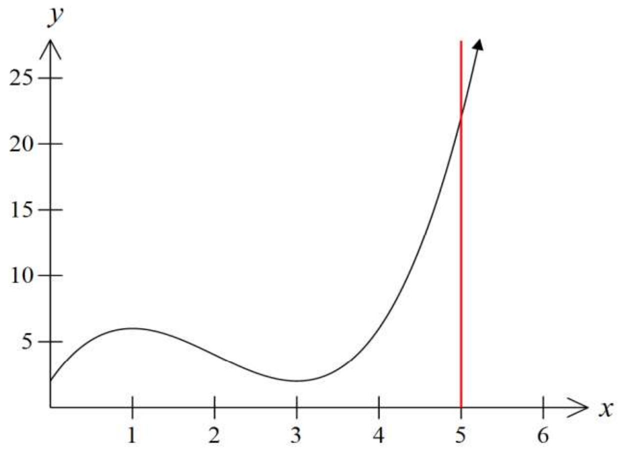
A. 70%

B. 16%

C. 54%

D. 48%

6 The diagram below shows the graph of $y = x^3 - 6x^2 + 9x + 2$ and the line $x = 5$.



What is the value of the area bounded by the x-axis and the curve $y = x^3 - 6x^2 + 9x + 2$ between $0 \leq x \leq 5$?

A. $A = 22.00$ square units

B. $A = 25.25$ square units

C. $A = 27.00$ square units

D. $A = 28.75$ square units

$A = \int_0^5 x^3 - 6x^2 + 9x + 2\,dx$

$= \left[\frac{x^4}{4} - 2x^3 + \frac{9x^2}{2} + 2x\right]_0^5$

$= \left[\frac{5^4}{4} - 2(5)^3 + \frac{9(5)^2}{2} + 2(5)\right] - 0$

$= 28.75\ u^2$

7 What is the amplitude, phase and period for the function $f(x) = 4\sin\left(\frac{x+\pi}{3}\right)$?

A. Amplitude 3, phase $\frac{\pi}{3}$ and period $\frac{\pi}{2}$

B. Amplitude 3, phase π and period $\frac{\pi}{2}$

C. Amplitude 4, phase $\frac{\pi}{3}$ and period 6π

D. Amplitude 4, phase π and period 6π

$f(x) = 4\sin\frac{1}{3}(x+\pi)$

Period $= \dfrac{2\pi}{1/3}$

$= 6\pi$

Amplitude is 4

Phase is π.

8 Let $g(x) = 1 - f(x)$. If $\int_1^4 f(x)dx = 7$, what is the value of $\int_1^4 g(x)dx$?

A. -6

B. -4

C. 8

D. 10

$\int_1^4 g(x)\,dx = \int_1^4 1 - f(x)\,dx$

$= \int_1^4 1\,dx - \int_1^4 f(x)\,dx$

$= x\Big]_1^4 - 7$

$= 4 - 1 - 7$

$= -4$

9 The curve $y = 2x^3 - ax^2 + 6x$ has a point of inflection point at x $= 4$.
What is the value of a?

A. -18

B. -24

C. 24

D. 18

$y = 2x^3 - ax^2 + 6x$

$y' = 6x^2 - 2ax + 6$

$y'' = 12x - 2a$

$y'' = 0$ $12x - 2a = 0$

when $x = 4$

$48 = 2a$

$a = 24$

10 The discrete random variable X has the following probability distribution.

x	-1	0	1	2	3
$P(X = x)$	$\frac{1}{10}$	a	b	0.3	$2b$

What are the values of a and b if the expected value $E(X)$ is 1.2?

A. $a = 0.3$ and $b = 0.1$

B. $a = 0.1$ and $b = 0.3$

C. $a = 0.3$ and $b = 0.3$

D. $a = 0.1$ and $b = 0.1$

$-1 \times \frac{1}{10} + 0 + b + 0.6 + 6b = 1.2$

$0.5 + 7b = 1.2$

$7b = 0.7$

$b = 0.1$

$\frac{1}{10} + a + b + 0.3 + 2b = 1$

$0.1 + a + 0.1 + 0.3 + 2(0.1) = 1$

$a = 0.3$

End of Section I

Mathematics Advanced

Section II Answer Booklet 1

Student Number:
Teacher:

Section II
90 marks
Attempt Questions 11 – 32
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Question 11 – 15 (15 marks)
Booklet 2 – Attempt Question 16 – 20 (15 marks)
Booklet 3 – Attempt Question 21 – 23 (15 marks)
Booklet 4 – Attempt Question 24 – 26 (17 marks)
Booklet 5 – Attempt Question 27 – 30 (15 marks)
Booklet 6 – Attempt Question 31 – 32 (13 marks)

Instructions

- Write your Teacher's Name and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 11 of Booklet 1. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (1 mark)

Find the derivative of $f(x) = 2x^2 - \frac{3}{x}$ 1

$f(x) = 2x^2 - 3x^{-1}$

$f'(x) = 4x + 3x^{-2}$

or $= 4x + \frac{3}{x^2}$

Question 12 (3 marks)

Calculate the sum of the arithmetic series $6 + 14 + 22 + \cdots + 958$. 3

$a = 6$, $d = 14-6 = 8$, $T_n = 958$

$T_n = a + (n-1)d$

$958 = 6 + (n-1) \times 8$

$958 = 6 + 8n - 8$

$958 = 8n - 2$

$960 = 8n$

$n = 120$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{120}{2}[2 \times 6 + 119 \times 8]$

$= 60[12 + 952]$

$= 57\,840$

Question 13 (4 marks)

Find

a) $\frac{d}{dx}(\ln\sqrt{3-x})$ 2

$$\frac{d}{dx}\left(\ln(3-x)^{\frac{1}{2}}\right) = \frac{d}{dx}\left(\frac{1}{2}\ln(3-x)\right)$$ 1

$$= \frac{1}{2} \times \frac{-1}{3-x}$$

$$= \frac{-1}{6-2x}$$ 1

b) $\frac{d}{dx}\left(\frac{\tan 3x}{x^3}\right)$ 2

$$\frac{d}{dx}\left(\frac{\tan 3x}{x^3}\right) = \frac{x^3 \cdot 3\sec^2 3x - \tan 3x \times 3x^2}{(x^3)^2}$$ — 2

$$= \frac{3x^3\sec^2 3x - 3x^2\tan 3x}{x^6}$$

$$= \frac{3\cancel{x^2}(x\sec^2 3x - \tan 3x)}{x^{\cancel{6}4}}$$

$$= \frac{3(x\sec^2 3x - \tan 3x)}{x^4}$$

Question 14 (2 marks)

Prove that $\cos\theta\cot\theta + \sin\theta = \operatorname{cosec}\theta$. 2

$$LHS = \cos\theta\cot\theta + \sin\theta$$

$$= \cos\theta \times \frac{\cos\theta}{\sin\theta} + \sin\theta$$ 1

$$= \frac{\cos^2\theta}{\sin\theta} + \sin\theta$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$$ 1

$$= \frac{1}{\sin\theta}$$

$$= \operatorname{cosec}\theta$$

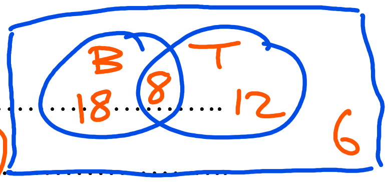
$$= RHS$$

Question 15 (5 marks)

Basketball and tennis are two sports students may choose to play.
For a group of 44 students, the following is known.

- 8 students play both.
- 20 students play tennis.
- 6 play neither.

a) A student is chosen at random. By using a Venn diagram or otherwise, find the probability that a student plays only basketball. **2**



- $20 - 8 = 12$ play tennis but not basketball
- $8 + 12 + 6 = 26 \rightarrow$ play basketball 1

$\therefore$ $44 - 26 = 18$ play basketball only 1

$\therefore P(\text{Basketball only}) = \frac{18}{44} = \frac{9}{22}$

b) A student is chosen at random. Given the student does not play basketball, what is the probability that the student plays tennis? **1**

$6 + 12 = 18$ students do not play basketball.

$\therefore P(\text{tennis} \mid \text{no basketball}) = \frac{12}{18}$

$= 2/3$

c) Two students from this group are randomly selected after each other. What is the probability that the first only plays basketball and the second plays only tennis? **2**

Probability first player plays basketball only $= \frac{9}{22}$

Probability Second player plays tennis only $= \frac{12}{43}$ 1

$\therefore P(B_1 \text{ and } T_2) = \frac{9}{22} \times \frac{12}{43}$

$= \frac{54}{473}$ 1

Question 16 (2 marks)

Given that $0 < x < 4$ and $\log_5(4 - x) - \log_5 3x = 1$, find the value of x. 2

$$\log_5(4-x) - \log_5 3x = 1$$

$$\log_5\left(\frac{4-x}{3x}\right) = 1$$ 1

$$\frac{4-x}{3x} = 5^1$$

$$\frac{4-x}{3x} = 5$$

$$4 - x = 15x$$

$$4 = 16x$$

$$x = \frac{1}{4}$$ 1

Question 17 (3 marks)

Evaluate $\int_2^3 \frac{x-3}{x^2-6x+5}\,dx$ 3

$$\frac{1}{2}\int \frac{2(x-3)}{x^2-6x+5}\,dx$$

$$= \frac{1}{2}\left[\ln|x^2 - 6x + 5|\right]_2^3$$ 1

$$= \frac{1}{2}\left[\ln|3^2 - 6(3) + 5| - \ln|2^2 - 6(2) + 5|\right]$$

$$= \frac{1}{2}\left[\ln|-4| - \ln|-3|\right]$$ 1

$$= \frac{1}{2}(\ln 4 - \ln 3)$$

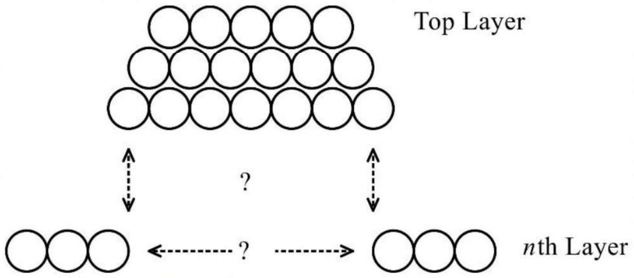
$$= \frac{1}{2}\ln\left(\frac{4}{3}\right)$$ 1

$$\doteqdot 0.1438\ldots$$

Question 18 (4 marks)

Marley works in a local fruit and vegetable store, and he is making a stack of oranges against a sloping display panel.

The oranges are stacked in layers, as shown, where each layer contains one orange less than the layer below it.



When he has finished, there are five oranges in the top layer, six in the next and so on.

There are n layers altogether.

a) Show there are $\frac{1}{2}n(n+9)$ oranges in the stack. **2**

Series is $5+6+7+8+\cdots+n$

$a=5 \quad d=6-5=1 \quad S_n\ ?$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{n}{2}[10+(n-1)\times 1]$

$\therefore S_n = \frac{n}{2}(9+n) = \frac{1}{2}n(n+9)$ as required.

b) If Lachlan has 300 oranges to create his display, how many full rows can he create, if the top row still contains five oranges? **2**

$S_n = \frac{1}{2}n(9+n)$

$300 = \frac{1}{2}n(9+n)$

$600 = n^2+9n$

$n^2+9n-600=0$

$n = \frac{-9 \pm \sqrt{81-4\times1\times-600}}{2}$

$= 20.4$ or -29.4 (not possible)

$\therefore$ Marley can make 20 complete rows.

Question 19 (3 marks)

a) Differentiate $(1 + \ln x)^4$. 2

$\frac{d}{dx}(1+\ln x)^4 = 4(1+\ln x)^3 \times \frac{1}{x}$ 2

$= \frac{4(1+\ln x)^3}{x}$

b) Hence, find $\int \frac{(1+\ln x)^3}{x}\,dx$. 1

$\int \left(\frac{1+\ln x)^3}{x}\right) = \frac{1}{4}\int \frac{4(1+\ln x)^3}{x}\,dx$

$= \frac{1}{4}(1+\ln x)^4 + c$ from (a).

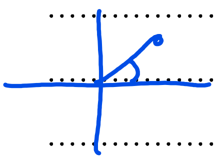
Question 20 (3 marks)

Solve the equation $2\sin^2\left(\frac{x}{3}\right) = 1$ where $[-\pi, \pi]$. 3

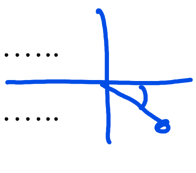
$\sin^2\left(\frac{x}{3}\right) = \frac{1}{2}$ for $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

$\sin\frac{x}{3} = \pm\frac{1}{\sqrt{2}}$

For $\sin\frac{x}{3} = \frac{1}{\sqrt{2}}$ or $\sin\frac{x}{3} = -\frac{1}{\sqrt{3}}$



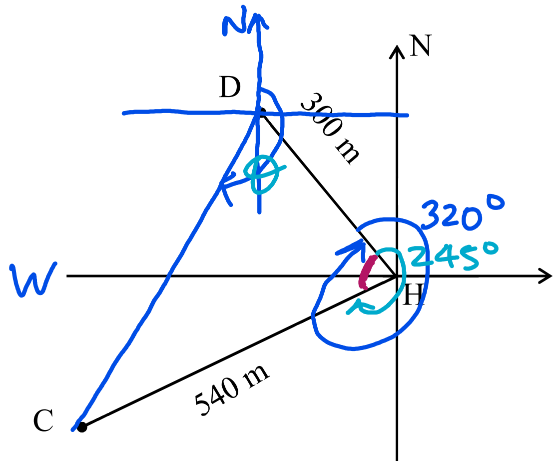
$\frac{x}{3} = \frac{\pi}{4}$ $\frac{x}{3} = -\frac{\pi}{4}$



$x = \frac{3\pi}{4}$ or $x = -\frac{3\pi}{4}$

Question 21 (5 marks)

Layla and her brother Mike are at their home H. Layla walked 300 m on a bearing 320° to her dancing school D and Mike walked 540 m on a bearing 245° to his sports club C.



a) Show that the angle DHC is 75°. 1

$\angle NHD = 40°$

$\therefore \angle DHC = 320 - 270 = 50°$

and $\angle CHW = 270 - 245 = 25°$

$\therefore \angle DHC = 50 + 25$
$= 75°$

1

b) Find the distance DC, correct to 2 decimal places. 2

$DC^2 = DH^2 + HC^2 - 2(DH)(HC)\cos\angle DHC$
$= 300^2 + 540^2 - 2(300)(540)\cos 75$
$= 297\,742.6294\ldots$

1

$\therefore DC = \sqrt{297742.6294\ldots}$
$= 545.657978\ldots$
$\doteqdot 545.66$ m (to 2 dp)

1

c) Find the bearing of the sports club C from the dancing school D correct to the nearest degree. 2

Let $\angle HDC = \theta$

$$\cos\theta = \frac{300^2 + 544.66^2 - 540^2}{2 \times 300 \times 544.66}$$

$= 0.29086..$

$\theta = \cos^{-1}(0.29086...)$

$= 73.922...^\circ$

$\therefore \angle HDC \doteqdot 73.922..^\circ$

Now $\angle HDN = 140^\circ$ (co-int angles add up to 180° on parallel lines)

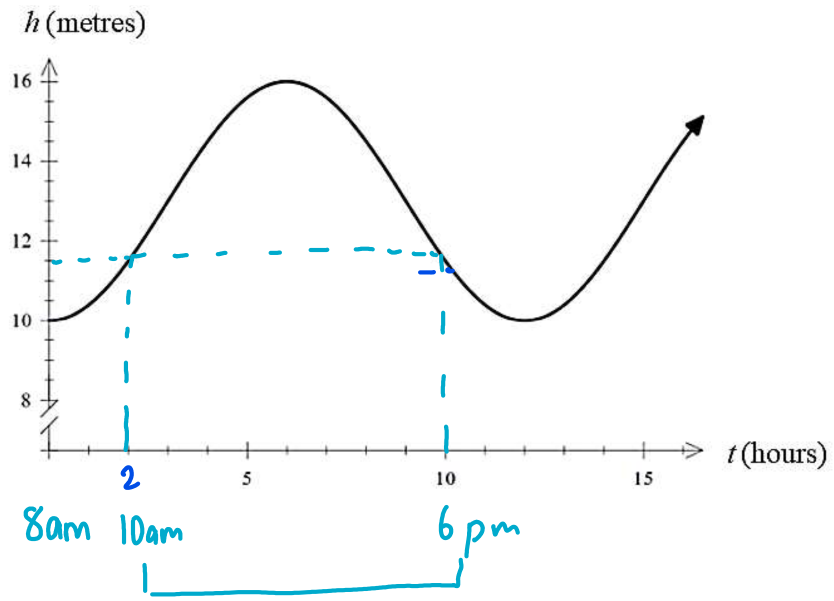
$\therefore$ the bearing of C from D is $140 + 73.922..$

$= 213^\circ T$ (to the nearest degree)

Question 22 (7 marks)

The rise and fall in sea level in Korsachov, due to tides, can be modelled by the cosine function below:

$$h(t) = A\cos(bt) + d$$



At 8am it is low tide, and the channel is 10m deep. At 2pm it is high tide, and the channel is 16m deep. A ship needs at least 11.5m of water to sail.

a) Find the value of A, b, and d. 3

$\boxed{A = -3}$

Period $= 12$

$\frac{2\pi}{b} = 12$

$2\pi = 12b$

$b = \frac{2\pi}{12}$

$\therefore \boxed{b = \frac{\pi}{6}}$

Centre of oscillation

$= \frac{16+10}{2} = 13$

$\therefore \boxed{d = 13}$

b) Between what times in the first day can the ship sail? 2

Now $h(t) = -3\cos\left(\frac{\pi t}{6}\right) + 13$

If $h = 11.5$ then

$11.5 = -3\cos\left(\frac{\pi}{6}t\right) + 13$

$\frac{-1.5}{-3} = \cos\left(\frac{\pi}{6}t\right)$

$\cos\left(\frac{\pi}{6}t\right) = \frac{1}{2}$

$\frac{\pi}{6}t = \frac{\pi}{3}, \frac{5\pi}{3}, \quad t = 2, 10h$

$\therefore$ The ship can sail between 10 am and 6pm

c) What is the rate of change of the water level at 12.00 p.m.? 2

$h'(t) = -3 \times \frac{\pi}{6}\left(-\sin\left(\frac{\pi t}{6}\right)\right)$

$= \frac{\pi}{2}\sin\frac{\pi t}{6}$

Now $h'(4) = \frac{\pi}{2}\sin\frac{\pi}{6}(4)$

$= \frac{\pi}{2}\sin\frac{2\pi}{3}$

$= \frac{\pi}{2} \times \frac{\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{4}$

$\therefore$ the level of water increases at a rate of $\frac{\pi\sqrt{3}}{4}$ m/h $\doteqdot 1.36$ m/h

Question 23 (3 marks)

Consider the geometric series

$$1 + (\sqrt{7} - 2) + (\sqrt{7} - 2)^2 + \cdots$$

a) Show with calculations why this geometric series has a limiting sum. 1

$r = \frac{T_2}{T_1} = \frac{\sqrt{7}-2}{1}$

$\doteqdot 0.6$

since $|r|<1$

$\therefore$ the series has a limiting sum.

b) Find the exact value of the limiting sum with a rational denominator. 2

$S = \frac{a}{1-r}$

$= \frac{1}{1-(\sqrt{7}-2)}$

$= \frac{1}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$

$= \frac{3+\sqrt{7}}{9-7}$

$= \frac{3+\sqrt{7}}{2}$

Question 24 (8 marks)

A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T°C, t minutes after it enters the liquid is given by,

$$T = 400e^{-0.05t} + 25, \quad t \geq 0.$$

a) Find the initial temperature of the ball as it enters the liquid. 1

$t=0$,

$T = 400e^{-0.05(0)} + 25$

$= 400 + 25$

$= 425$

$\therefore$ the initial temperature is 425°.

b) Find the value of t for which $T = 200$, giving you answer to 3 decimal places. 2

$200 = 400e^{-0.05t} + 25$

$\frac{175}{400} = e^{-0.05t}$

$\ln \frac{175}{400} = -0.05t$

$t = \frac{\ln \frac{175}{400}}{-0.05}$

$= 16.53357146$ $\therefore t = 16.534$ min

c) Find the rate at which the temperature of the ball is decreasing after 50 minutes, giving your answer to 3 decimal places. 2

$\frac{dT}{dt} = -20e^{-0.05t}$

when $t = 50$

$\frac{dT}{dt} = -20e^{-0.05(50)}$

$= -1.642...$

$\therefore$ temperature is decreasing at a rate of 1.642°C/min.

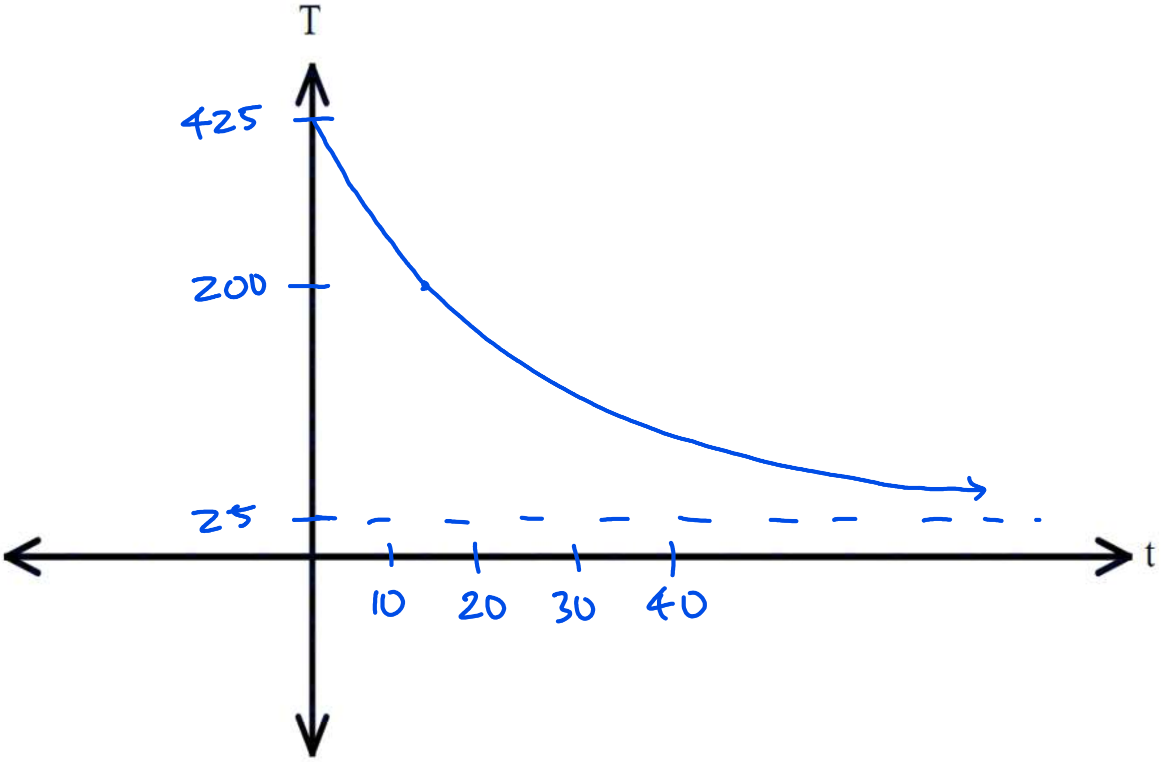
d) As it cools the ball will eventually approach what temperature? **1**

As $t \to \infty$

$400e^{-0.05t} \to 0$

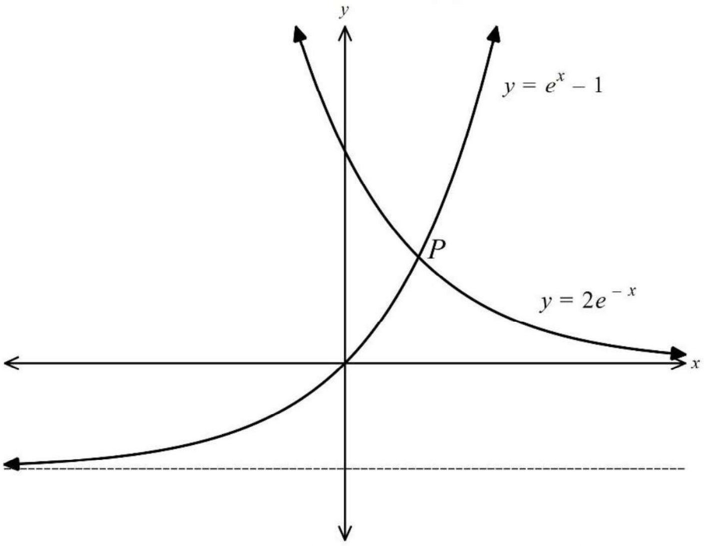
$\therefore T \to 25$

e) Sketch the graph of T. 2



Question 25 (6 marks)

Two curves $y = 2e^{-x}$ and $y = e^x - 1$ intersect at a point P.



a) Show algebraically that the coordinates of P are $(\ln 2, 1)$. 3

$y = 2e^{-x}$ ---- ①

$y = e^x - 1$ ..... ②

① = ②

$2e^{-x} = e^x - 1$

$\frac{2}{e^x} = e^x - 1$

$2 = e^{2x} - e^x$ 1

$e^{2x} - e^x - 2 = 0$

Let $m = e^x$

$m^2 - m - 2 = 0$

$(m-2)(m+1) = 0$

$\therefore m = 2$ or $m = -1$

$e^x = 2$ $\quad$ $e^x = -1$ but $e^x \neq 0$ 1

$\ln e^x = \ln 2$ $\quad$ $\therefore$ no solution

$x = \ln 2$

when $x = \ln 2$, $y = 2 \cdot e^{-\ln 2}$ 1

$= 2 \times \frac{1}{2} = 1$

$\therefore$ Intersects at $(\ln 2, 1)$

b) Find the area bounded by the two curves and the y-axis. 3

$$A = \int_0^{\ln 2} 2e^{-x}\,dx - \int_0^{\ln 2} e^x - 1\,dx$$

$$= \left[-2e^{-x}\right]_0^{\ln 2} - \left[e^x - x\right]_0^{\ln 2}$$

$$= \left[-2e^{-\ln 2} - (-2e^0)\right] - \left[e^{\ln 2} - \ln 2 - (e^0 - 0)\right]$$

$$= \left(-2 \times \tfrac{1}{2} + 2\right) - \left(2 - \ln 2 - 1\right)$$

$$= 1 - (1 - \ln 2)$$

$$= \ln 2$$

Question 26 (3 marks)

The parabola $y = x^2 - 2x$ is dilated horizontally by factor of 2 then translated vertically up by 3 to give a new parabola.

a) Sketch the transformed new parabola showing its vertex. 2

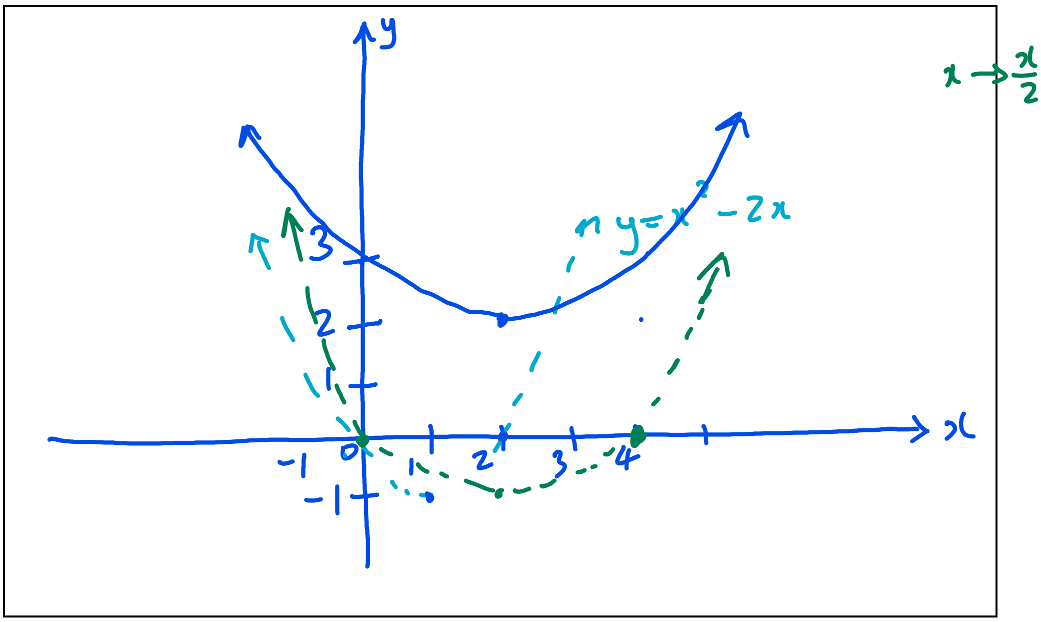
$y = x^2 - 2x$

Horizontally dilated by factor of 2, $x \to x/2$

Vertical translated up by 3, $y \to y - 3$

For $y = x^2 - 2x$
$= x(x-2)$

Vertex is $(1, -1)$ $\therefore$ New vertex is $(2, 2)$



b) Find the equation of the new transformed parabola. 1

From $y = x^2 - 2x$

i) $x \to \frac{x}{2}$, $y = \left(\frac{x}{2}\right)^2 - 2\left(\frac{x}{2}\right)$

$= \frac{x^2}{4} - x$

$y \to y - 3$ $y - 3 = \frac{x^2}{4} - x$

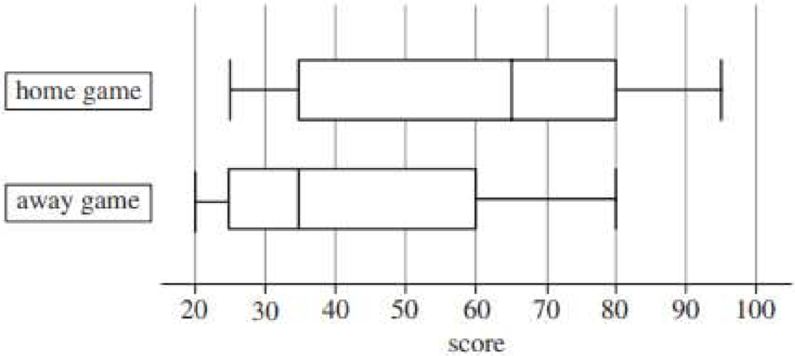
$y = \frac{x^2}{4} - x + 3$

Question 27 (5 marks)

A local basketball team played 23 home games and 23 away games.

The home game (H) scores are the scores that the team made during games at their own basketball court; the away game (A) scores are the scores that the team made during games at opponents' courts.

The scores from each game are represented in the box-plot.



a) What was the team's lowest home game score? 1

25

b) What percentage of the team's away game scores were less than 35? 1

50%

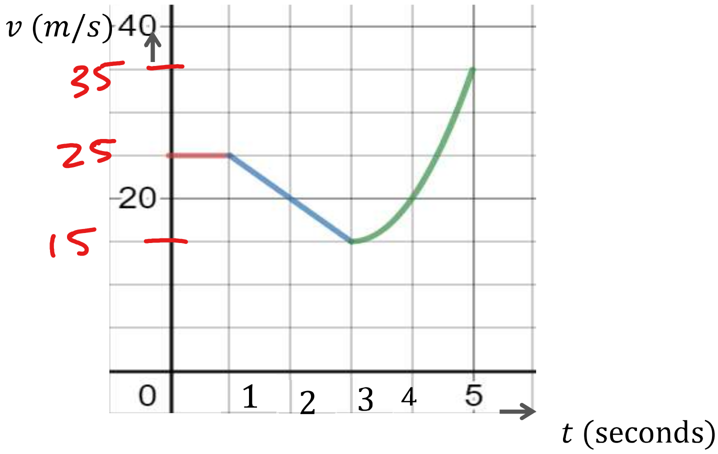
c) Compare and contrast the distributions of the team's home and away game scores. 3
In your answer, comment on the skewness of the distributions, measures of central tendency and spread.

- Home game scores are negatively skewed
  Away game " " positively skewed
- Home game (H) scores have a higher median (= 65) than the Away game (A) scores (with a median 35)
- For H → IQR = 80 − 35 = 45
  range = 70
  For A → IQR = 60 − 25 = 35 and range =

Although Home games have higher scores there is more consistency and less spread in the Away games

Question 28 (4 marks)

The graph shows the velocity (in metres per second) of a particle for 5 seconds.



The distance covered by the particle over 5 seconds is given by $\int_0^5 v(t)\ dt$.

Use the trapezoidal rule and the velocity at each of the six times values from $t = 0$ to $t = 5$ to find the approximate distance covered.

Give your answer correct to one decimal place. 2

With the velocity-time graph, the area under the curve gives the distance travelled by the particle, so using the Trapezoidal rule:

$A \doteqdot \frac{5-0}{\cancel{5}}\!\!\!\!\!\!\quad\frac{}{2}\left[25 + 35 + 2(25+20+15+20)\right]$

$\doteqdot 110$

$\therefore$ Approx, dist. travelled is 110m.

b) Is the distance travelled between 3 seconds and 5 seconds an overestimate or an underestimate of the actual distance covered by the particle? Justify your answer. 2

The distance travelled between 3 sec and 5 sec is an overestimate as as the curve is concave up.

Question 29 (4 marks)

The sum of the first two terms of an infinite geometric series is 45. The third term in the series is 12. 4

Show that there are two possible series and find the first term and common ratio for each case.

Let $T_1 = a$ be first term

and $T_2 = ar$ be the 2nd term of a GP

Now $T_1 + T_2 = 45$

$a + ar = 45$

$a(1+r) = 45 \quad \text{----} (1)$

Also $T_3 = 12$

$ar^2 = 12$

$a = \frac{12}{r^2}$ sub in (1)

$\frac{12}{r^2}(1+r) = 45$

$12 + 12r = 45r^2$

$45r^2 - 12r - 12 = 0$

$15r^2 - 4r - 4 = 0$

$r = \frac{4 \pm \sqrt{16 - 4 \times 15 \times -4}}{2 \times 15}$

$= \frac{4 \pm \sqrt{256}}{30}$

$= \frac{4 \pm 16}{30}$

$r = \frac{4+16}{30}$ or $\frac{4-16}{30}$

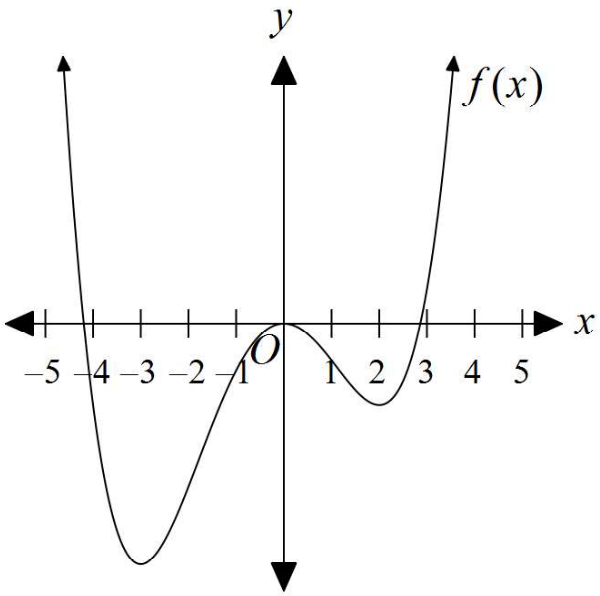
$r = \frac{2}{3}$ or $-\frac{2}{5}$

when $r = \frac{2}{3}$ sub in (2) when $r = -\frac{2}{5}$ sub in (2)

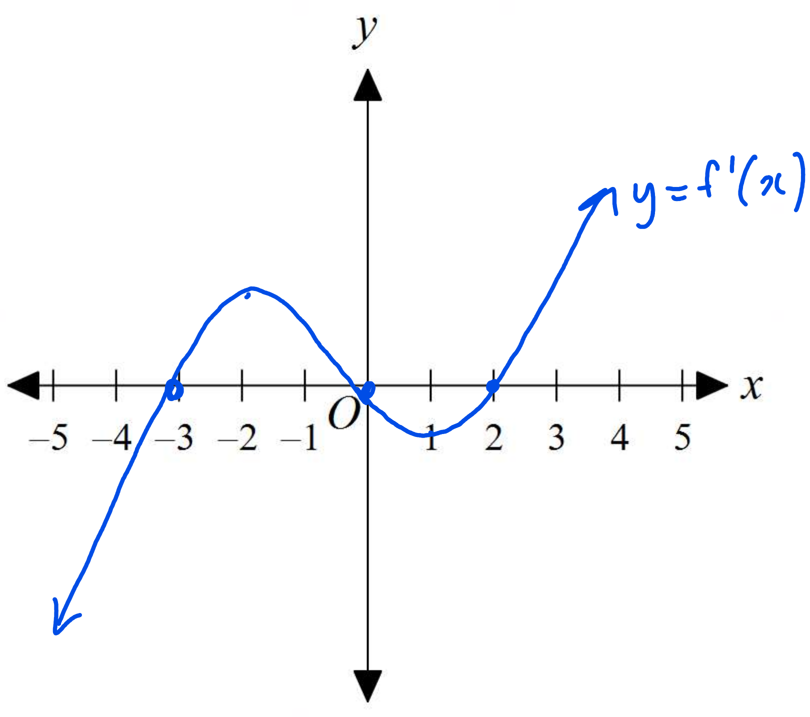
$a = 27$ $\therefore a = 75$

Question 30 (2 marks)

The diagram shows the graph of $f(x)$.



Sketch the graph of $f'(x)$, showing approximate $x-$intercepts and the behaviour as $x \to \pm\infty$. 2



Question 31 (6 marks)

A particle is moving in a straight line with a velocity given by $\frac{dx}{dt} = 8\cos\left(2t - \frac{\pi}{2}\right)$,

where x is the displacement from the origin in metres and t is time measured in seconds. The particle is initially at rest 4 m to the right of the origin.

a) Show that the displacement of the particle is given by $x = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$. 2

$x = \int 8\cos\left(2t - \frac{\pi}{2}\right) dt$

$= 8\int \cos\left(2t - \frac{\pi}{2}\right) dt$

$= \frac{8}{2}\sin\left(2t - \frac{\pi}{2}\right) + c$

$= 4\sin\left(2t - \frac{\pi}{2}\right) + c$

when $t=0$, $x=4$, $4 = 4\sin\left(2(0) - \pi/2\right) + c$

$4 = 4(-1) + c$

$c = 8$ $\therefore \boxed{x = 4\sin\left(2t - \frac{\pi}{2}\right) + 8}$

b) Find the first time when the particle comes to rest again. 2

Particle comes to rest when $\frac{dx}{dt} = 0$

$8\cos\left(2t - \frac{\pi}{2}\right) = 0$

$\cos\left(2t - \frac{\pi}{2}\right) = 0$

$2t - \frac{\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$2t = \pi, 2\pi, \ldots$

$t = \pi/2, \pi, \ldots$

$\therefore$ The particle comes to rest again after $\pi/2$ seconds.

c) By finding a function for the acceleration, describe the motion of the particle directly after $\frac{\pi}{2}$ s and before it next comes to rest. 2

$\frac{d^2x}{dt^2} = -16\sin\left(2t - \frac{\pi}{2}\right)$

when $t = \frac{\pi}{2}$, $\frac{d^2x}{dt^2} = -16\sin\left(2(\pi/2) - \pi/2\right)$

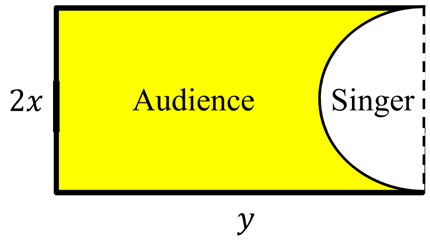
$= -16\sin \pi/2$

$= -16$

$\therefore$ the particle will start moving towards the left after being stationary at $\pi/2$ sec.

Question 32 (7 marks)

A concert hall is to be built in the shape of a rectangle with width $2x$ metres and length y metres.



The hall is to be divided into two parts. One section is for the audience and the other section is for the singer as shown in the diagram above.

The audience section is required to have an area of 500 m^2 and its perimeter should be as small as possible.

Let P be the perimeter of the audience section of the concert hall.

a) Show that the perimeter of the audience section of the concert hall is given by $P = \left(2 + \frac{3\pi}{2}\right)x + \frac{500}{x}$. 3

Area of audience $= 2x \times y - \left(\frac{\pi x^2}{2}\right)$

but $A_{audience} = 500$ (given)

$\therefore\ 500 = 2xy - \frac{\pi x^2}{2}$

$2xy = 500 + \frac{\pi x^2}{2}$

$y = \frac{500}{2x} + \frac{\pi x^2}{4x}$

$\therefore\ y = \frac{250}{x} + \frac{\pi x^2}{4x}$ --- (1)

Perimeter, $P = 2x + 2y + \pi x$

$= 2x + 2\left(\frac{250}{x} + \frac{\pi x^2}{4x}\right) + \pi x$ from (1)

$= 2x + \frac{500}{x} + \frac{\pi x}{2} + \pi x$

$= 2x + \frac{500}{x} + \frac{3\pi x}{2}$

$= \left(2 + \frac{3\pi}{2}\right)x + \frac{500}{x}$ #

b) Find the radius of the semicircle of the hall where the singer stands that gives the smallest possible perimeter for the audience section. 4

Show, by mathematical reasoning, why this radius gives the required minimum perimeter.

$$P = \left(2 + \frac{3\pi}{2}\right)x + \frac{500}{x}$$

$$\frac{dP}{dx} = 2 + \frac{3\pi}{2} - \frac{500}{x^2}$$

Min perimeter will occur when $\frac{dP}{dx} = 0$

i.e $2 + \frac{3\pi}{2} - \frac{500}{x^2} = 0$

$$4 + 3\pi - \frac{1000}{x^2} = 0$$

$$\frac{1000}{x^2} = 4 + 3\pi$$

$$\frac{x^2}{1000} = \frac{1}{4 + 3\pi}$$

$$x^2 = \frac{1000}{4 + 3\pi}$$

$x = \sqrt{\frac{1000}{4 + 3\pi}}$ as $x > 0$.

Now $\frac{d^2P}{dx^2} = \frac{1000}{x^3}$

when $x = \sqrt{\frac{1000}{4 + 3\pi}}$

$$\frac{d^2P}{dx^2} = \frac{1000}{\left(\sqrt{\frac{1000}{4+3\pi}}\right)^3} \doteqdot 197.476 > 0$$

$\therefore$ concave up

$\therefore$ minimum perimeter occurs when $x = \sqrt{\frac{1000}{4 + 3\pi}} \doteqdot 5.0639\ldots$ m

END OF EXAMINATION